# Discussion of "Notip: Non-parametric True Discovery Proportion control for brain imaging"

## by Alexandre Blain, Bertrand Thirion, Pierre Neuvial

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- Definition of $k_{\max}$ in the fMRI data framework: region with **high proportion of actived voxels guaranteed**.
- **Python implementation**, which is used a lot by neuroscientists.
- The paper is fully **reproducible**, which helps me to follow perfectly the computational analysis.

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BUT... remains parametric $\rightarrow$ do not capture the **dependence structure** of fMRI data.

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BUT.. the critical vector are Simes-based, i.e., linear shape $\rightarrow$ do not capture the **shape** of the null distribution, **sensitive** to the smallest p-values.

Finally... **Notip**!

# FDP upper bound

$$V^t(S) = \min_{1 \le k \le |S| \wedge k_{max}} \Big\{ \sum_{i \in S} 1\{p_i(X) \ge t_k\} + k - 1 \Big\}$$

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- **parametric** Simes-based $\rightarrow t_k = \dfrac{\alpha k}{m}$
- **permutation** Simes-based $\rightarrow t_k = \dfrac{\lambda_\alpha k}{m}$
- **permutation learned** template $\rightarrow t_k^b = \inf\{x : b/B \leq F_{p_k}(x)\}$

# Question 1

However, there are other templates beyond the Simes-based ones (*Blanchard (2008)*, *Hemerik (2019)*, ...). In particular:

- **Beta** family $\rightarrow t_k = \inf\{x : \lambda_\alpha \leq F_k(x)\}$ where $F_k(x)$ is the cumulative distribution function of $Beta(k, m+1-k)$.

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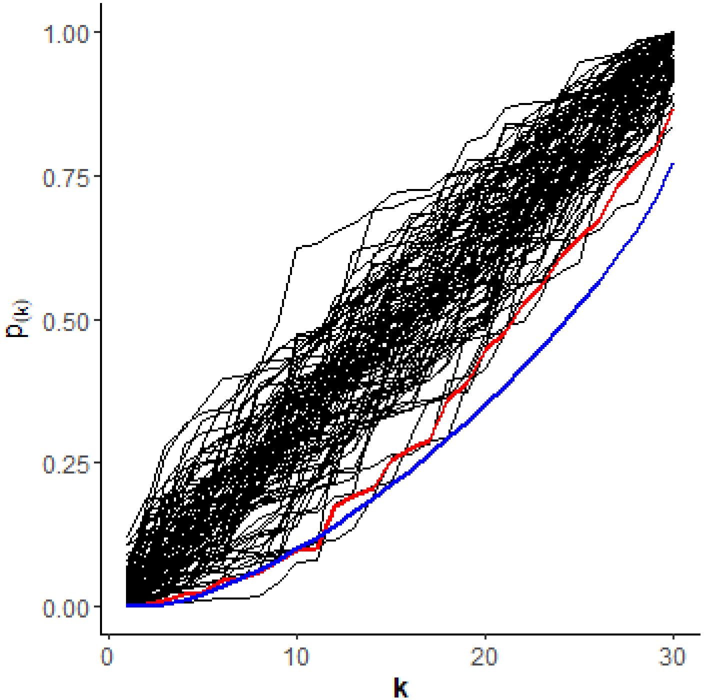


- Notip TDP lower bound: 43.8 $\%$,
- Beta TDP lower bound: 49.1 $\%$,

> **Question 1**: Did you try different types of templates, not Simes-based, in your simulations?

# Questions 2-3 (again about templates but in fMRI)

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- **Size of the cluster**
- **Structure of the null distribution**

For example, in your application the learned template seems to outperform the calibrated Simes in the case of **large cluster** (like the `sumsome` method from *Vesely (2021)*). Probably due to the conservativness of simes-based templates for the smallest p-values.

# Questions 2-3 (again about templates but in fMRI)

For that, *Hemerik (2019)* proposed a **shifted** version of the Simes-based family:

$$t_k = \frac{(k - \delta)\lambda}{m - \delta}$$

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> **Question 3**: How do you think the cluster width, signal strength, and conservative/anti-conservative structure of the null distribution affect the family choice in a given fMRI?

# Question 4

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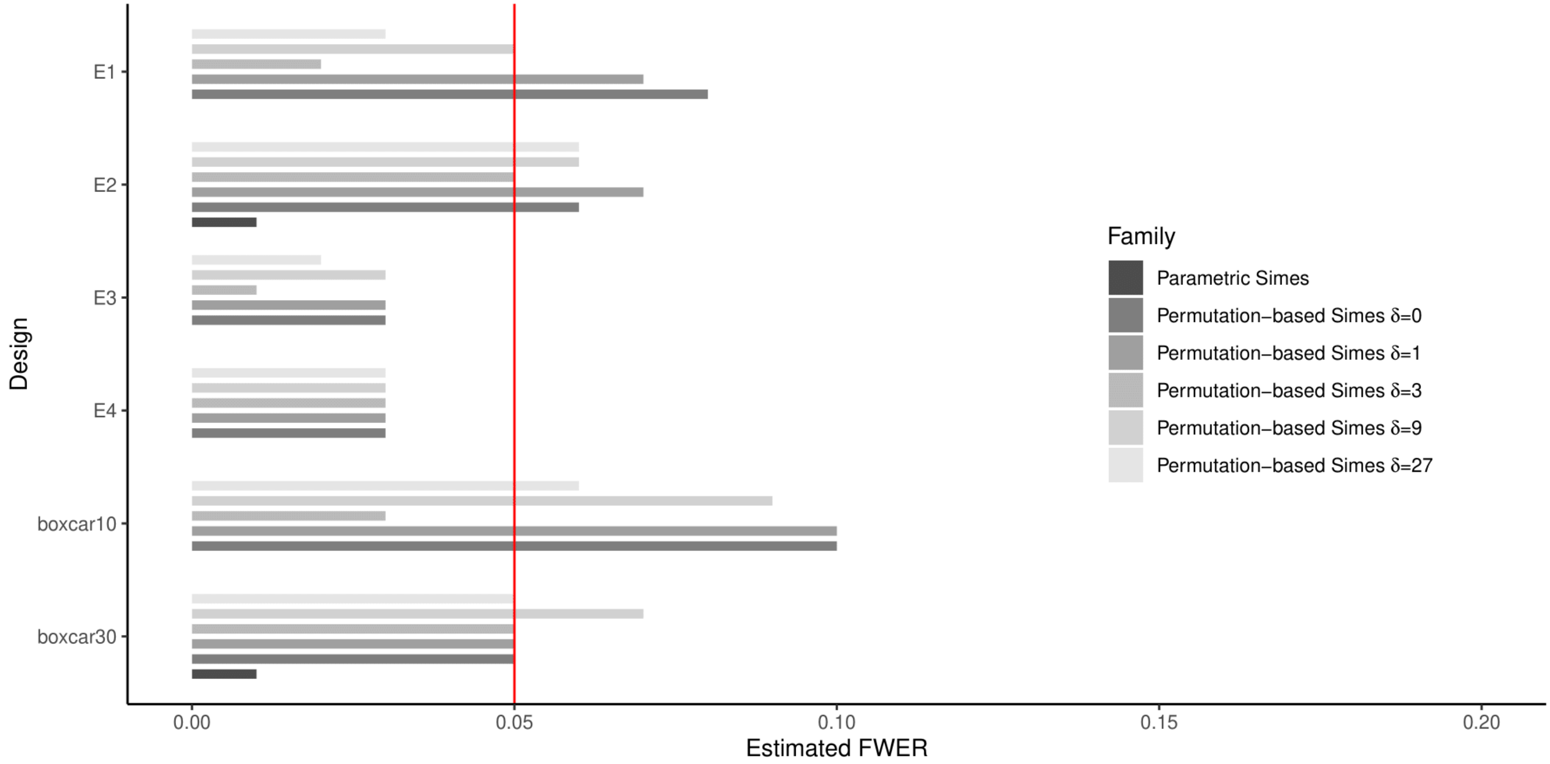

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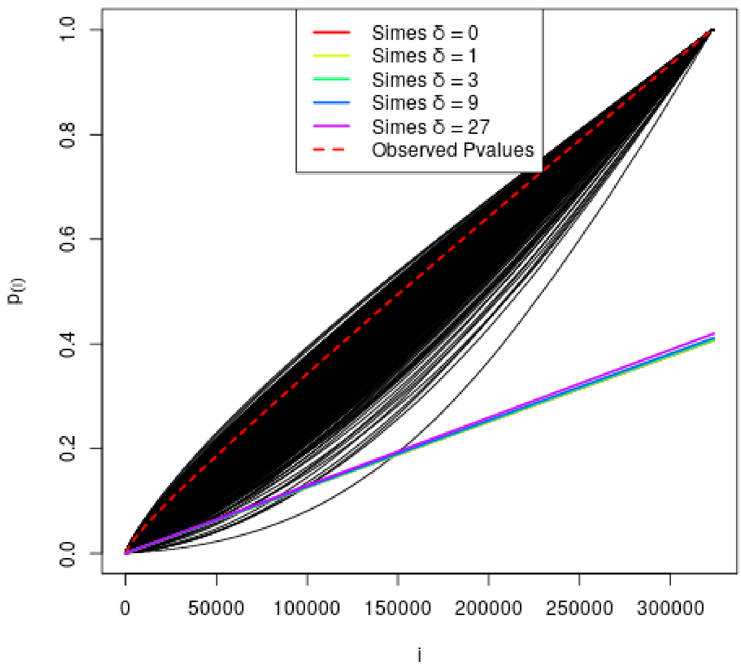
Simes δ = 0
Simes δ = 1
Simes δ = 3
Simes δ = 9
Simes δ = 27
Observed Pvalues
1.0
0.8
0.6
0.4
0.2
0.0
P(i)
0
50000
100000
150000
200000
250000
300000
i

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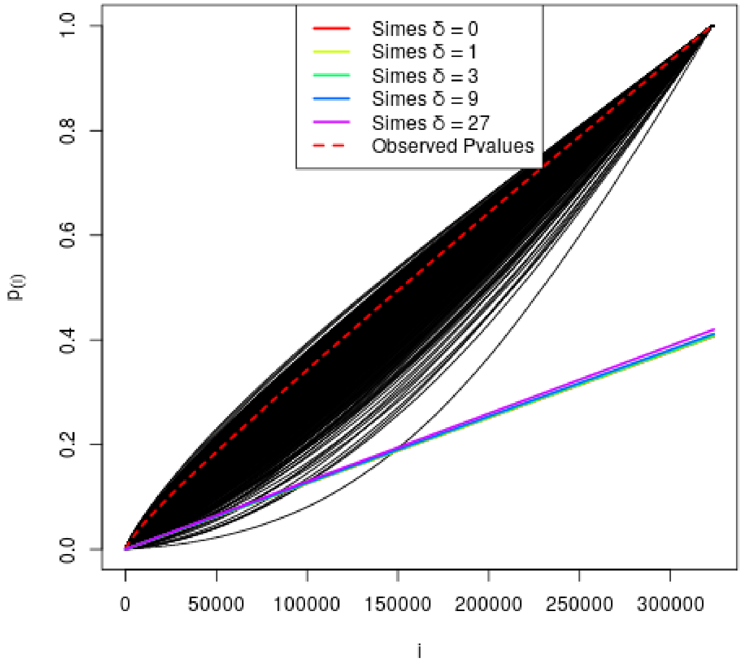


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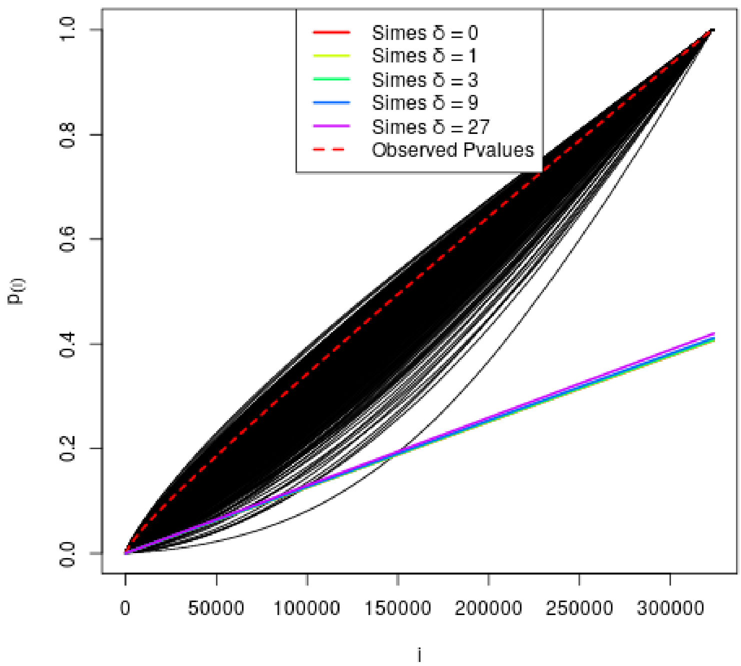


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There is probably difficulty in doing this analysis given the computational cost of Notip in learning the template (?).

# Recap questions

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> **Question 2**: Did you try the shifted version using Neurovault data?

> **Question 3**: How do you think the cluster width, signal strength, and conservative/anti-conservative structure of the null distribution affect the family choice in a given fMRI?

> **Question 4**: How do you think your template might react using resting state data?

# References

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